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.0000208$\overline{3}$ = error bound = $\left|\frac{.05^3}{3!}\right|$

Next Term $\frac{-x^3}{3!}$

$f(x)=\sin x$ $f'(x)=\cos x$ $f''(x)=-\sin x$ $f'''(x)=-\cos x$	9) Given that $P_1(x) = x$ represents the first order polynomial for sinx centered at x = 0. Use the Lagrange Error Bound to find the error when $\lvert x\rvert \le .05$ $-.05 \le x \le .05$ x = -.05: $\frac{f'''(-.05)\,x^3}{3!}$, $\frac{-\cos(-.05)x^3}{3!}$ x = 0: $\frac{f'''(0)\,x^3}{3!}$, $\frac{-1x^3}{3!}$ x = .05: $\frac{f'''(.05)\,x^3}{3!}$, $\frac{-\cos(.05)x^3}{3!}$
$f(x)=\ln x$ $f'(x)=\frac{1}{x}=x^{-1}$ $f''(x)=-x^{-2}$ $f'''(x)=2x^{-3}$ $f^4(x)=-6x^{-4}=\frac{-6}{x^4}$ Next Term $\left\lvert -\frac{1}{4}(x-1)^4\right\rvert \to \frac{1}{4}(.1)^4$ $=.000025$	14) Given that $P_3(x)=(x-1)-\frac{1}{2}(x-1)^2+\frac{1}{3}(x-1)^3$ represents the third order Taylor polynomial for ln(x) centered at x = 1. Use the Lagrange Error Bound to find the error when $\lvert x-1\rvert \le .1$ $-.1 \le x-1 \le .1$; $+1$ $+1$ $+1$; $+.9 \le x \le 1.1$ x = .9 x = 1 x = 1.1 Which of these has the biggest 4th derivative? $\left\lvert\frac{f^4(.9)(x-1)^4}{4!}\right\rvert = \left\lvert\frac{\frac{-6}{.9^4}(x-1)^4}{4!}\right\rvert$ Formula for Error Bound Distance from center $\frac{\frac{6}{.9^4}(.1)^4}{4!} = .0000381039476$

Summary of Error Bound

For an Alternating Series – Use the next term

For a series that is Not Alternating

1. Write down the formula for the next derivative.

2. Find the value of the next derivative at the ends of the interval and the center.

3. Whichever value is bigger is the value you use to build your error bound term